Solutions

Signs: (-2).

Math 2D Quiz 1 Morning - March 31, 2016

Please put your ID on back for redistribution!

Show all of your work. *There is a question on the back side.

1. [10pts] Evaluate the following integral:

$$\int_0^1 \int_1^2 (x + e^{-y})\, dx\, dy.$$

Simplify your answer.

$$\hookrightarrow \int_0^1 \left. \frac{x^2}{2} + xe^{-y} \right|_{x=1}^{x=2} dy = \int_0^1 \left[2 + 2e^{-y} - \left(\frac{1}{2} + e^{-y} \right) \right] dy$$

+5

$$= \int_0^1 \left(\frac{3}{2} + e^{-y} \right) dy = \left. \frac{3y}{2} - e^{-y} \right|_{y=0}^{y=1}$$

$$= \frac{3}{2} - e^{-1} - (0 - \underbrace{e^0}_{=1}) = \boxed{\frac{5}{2} - e^{-1}}$$ +5

If you applied Fubini, or to check:

$$\int_1^2 \int_0^1 (x + e^{-y})\, dy\, dx = \int_1^2 \left. xy - e^{-y} \right|_{y=0}^{y=1} dx$$

$$= \int_1^2 \left[(x - e^{-1}) - (0 - e^0) \right] dx$$

(+5)

$$= \int_1^2 (x - e^{-1} + 1)\, dx = \left. \frac{x^2}{2} - xe^{-1} + x \right|_{x=1}^{x=2}$$

$$= (2 - 2e^{-1} + 2) - \left(\frac{1}{2} - e^{-1} + 1 \right) = 4 - \frac{3}{2} - e^{-1}$$

$$= \boxed{\frac{5}{2} - e^{-1}}$$ (+5) ✓

2. [10pts] Evaluate the following integral:

$$\iint_R (x + xy^{-2})\, dA$$

where $R = \{(x, y) \in \mathbb{R}^2 : 0 \le x \le 2,\ 1 \le y \le 2\}$. Simplify your answer to a single number.

dxdy :

$$\int_{y=1}^{y=2} \int_{x=0}^{x=2} \left(x + \frac{x}{y^2}\right) dx\, dy = \int_1^2 \left(\frac{x^2}{2} + \frac{x^2}{2y^2}\right)\Bigg|_{x=0}^{x=2} dy$$

(+6)

$$= \int_1^2 \left[\left(2 + \frac{2}{y^2}\right) - 0 - 0\right] dy = 2y - \frac{2}{y}\Bigg|_{y=1}^{y=2}$$

$$= (4-1) - (2-2) = \boxed{3}$$ +5 +4

dydx :

$$\int_{x=0}^{x=2} \int_{y=1}^{y=2} \left(x + \frac{x}{y^2}\right) dy\, dx = \int_0^2 xy - \frac{x}{y}\Bigg|_{y=1}^{y=2} dx$$

$$= \int_0^2 \left[\left(2x - \frac{x}{2}\right) - (\cancel{x} - \cancel{x})\right] dx = \int_0^2 \frac{3x}{2}\, dx$$ +6

$$= \frac{3x^2}{4}\Bigg|_0^2 = \frac{12}{4} = \boxed{3}$$ +5 +4